A Refutation of Sallé’s Longstanding Conjecture

The Lambda Calculus . . . why ?
At the basis of all functional languages. Simple syntax but Turing complete. The lattice of \( \lambda \)-theories is a mathematically rich structure. Many links with mathematical logic: intuitionistic logic, linear logic, universal algebra, category theory, recursion theory . . .

Observational Equivalences \( H^+ \) and \( H^* \)

When are two programs \( M \) and \( N \) observationally equivalent?

The \( \lambda \)-theory

- \( H^+ \) corresponds to \( O := \) solvable \( \lambda \)-terms
- \( H^* \) corresponds to \( O := \beta \)-normalizable \( \lambda \)-terms

\( B \omega \) : Böhm Trees + Strong Extensionality

Böhm Trees

\[ B \vdash M = N \] \iff \[ \text{BT}(M) = \text{BT}(N) \]

The \( \omega \)-rule

\[ \forall P \in O^0 . MP = NP \implies M = N \]

Sallé’s Conjecture — 1979

Lambda Calculus Conference in Swansea 1979

The Refutation — 2017

The New Kite

The Recipe

Ingredients

- \( B \omega \subseteq H^+ \):
  - The characterisations of \( H^+, H^* \) in terms of extensional Böhm trees,
  - a weak semi-separation theorem,
  - Böhm-out technique (q.b.);

- \( H^+ \subseteq B \omega \):
  - Self-interpreters,
  - a pinch of recursion theory,
  - a key property of solvable terms.

An International Collaboration

- Breuvart
- Manzonetto
- Ruoppolo
- Intrigila
- Polonsky

The Kite of \( \lambda \)-Theories

The Unrefuted Conjecture

The Refutation

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