CALMOC: Categorical and Algebraic Models of Computation
Henk Barendregt, Mai Gehrke, Herman Geuvers, Giulio Manzonetto and Jan Rutten
Intelligent Systems, Radboud Universiteit, Nijmegen, The Nederlands

The Problem

- Non-determinism: understand and exploit true concurrency
  - extensive study of process calculi, trace semantics, relaxed memory models did not provide definitive answers.
- Program robustness: static check of dynamic properties
  - control at compile-time on the amount of resources needed by a program at running-time
- Optimization via program rewriting:
  - syntactic and semantic characterization of operational equivalence

Methodology

- Object of study: the resource calculus (Ehrhard-Regnier 2003)
  - functional programming language based on λ-calculus,
  - explicit handle on the resources used by a program during its execution (unlike Java or C).
- Abstract mathematical description of models of the resource calculus
  - definition of model based on category-theory / universal algebra.
- Mathematical tools for studying the resource calculus:
  - program decomposition through Taylor expansion (link with analysis),
  - natural duality theory for algebraic models (link with topology)
- Study of definability/adequacy/full abstraction on concrete models:
  - relational semantics: from functional to relational interpretations,
  - game semantics: from static to interactive denotations.

The Big Picture

- Denotational Semantics
  - full abstraction
- Categorical Models
- Algebraic Models
- Taylor Expansion
- Operational Semantics
- Resource Calculus
- Syntactical differentiation
- Explicit handle of resources

New Ingredient: Taylor Expansion of Programs

- Program differentiation:
  - add a syntactic derivative operator \( D(\cdot) \) computing the best linear approximation of a program,
  - excellent candidate to increase control over programs executed in environments with bounded resources.
- Taylor Expansion: replace the usual application \( P \cdot x \) of a program \( P \) to an input \( x \) by a series of linear applications
  \[
  P \cdot x = \sum_{n=0}^{\infty} \frac{1}{n!} (D^n P \cdot x^n)0
  \]
- Breakthrough: transfer results
  - from the linear fragment of the resource calculus \((simple)\)
  - to classic programming languages/full resource calculus \((complex)\).

Expected Results: Stone Duality for Algebraic Models

- Scott domains vs Stone spaces
- Stone duality builds a bridge between
  - Algebraic models of computation
  - Topological spaces

Expected Results: Characterization of Behavioural Equivalence

- Behavioural equivalence of two programs \( P_1 \) and \( P_2 \):
  - Syntactic approach \( P_1 =_o P_2 \): two programs are equivalent if they have the same behaviour in every context,
  - Logical approach \( P_1 =_r P_2 \): two programs are equivalent if they have the same interpretation in a model \( D \),
  - New approach \( P_1 =_T P_2 \): two programs are equivalent if they have “similar” Taylor expansions.
- Proof of equivalence of these approaches:
  - Syntactic Approach
  - Logical Approach

Expected Results: Resource Game Semantics

- Resource sensitive models based on game theory,
  - from static to dynamic and interactive interpretations of programs,
- Main ingredients:
  - 2-player games: player vs opponent,
  - alternating games,
  - plays satisfying well-bracketing,
  - non-deterministic strategies,
- Build and study a fully abstract model.

Applications

- Communications: study of programs running in environments with bounded resources (smartphones, PDA’s, etc.),
- Security: prevent run-time failures caused by memory limitations in critical fragments of code.
- Programming: design of new programming languages inspired from semantics,
- Fancy computer science: handle data that cannot be duplicated for physical reasons like q-bits in quantum programming.

Research Team

- Coordinators:
  - Henk Barendregt (PI)
    - ICIS, Radboud University
  - Mai Gehrke
    - IMAPP, Radboud University
  - Herman Geuvers
    - ICIS, Radboud University
  - Researchers
    - Giulio Manzonetto
      - ICIS, Radboud University
    - Jan Rutten
      - CWI & Radboud University

New Approach

- Syntactic approach
  - plays satisfying well-bracketing,
- Logical approach
  - two programs are equivalent if they have the same interpretation in a model \( D \),
- New approach
  - two programs are equivalent if they have “similar” Taylor expansions.

Expected Results: Stone Duality for Algebraic Models

- Scott domains vs Stone spaces
- Stone duality builds a bridge between
  - Algebraic models of computation
  - Topological spaces

Expected Results: Characterization of Behavioural Equivalence

- Behavioural equivalence of two programs \( P_1 \) and \( P_2 \):
  - Syntactic approach \( P_1 =_o P_2 \): two programs are equivalent if they have the same behaviour in every context,
  - Logical approach \( P_1 =_r P_2 \): two programs are equivalent if they have the same interpretation in a model \( D \),
  - New approach \( P_1 =_T P_2 \): two programs are equivalent if they have “similar” Taylor expansions.
- Proof of equivalence of these approaches:
  - Syntactic Approach
  - Logical Approach

Expected Results: Resource Game Semantics

- Resource sensitive models based on game theory,
  - from static to dynamic and interactive interpretations of programs,
- Main ingredients:
  - 2-player games: player vs opponent,
  - alternating games,
  - plays satisfying well-bracketing,
  - non-deterministic strategies,
- Build and study a fully abstract model.

Applications

- Communications: study of programs running in environments with bounded resources (smartphones, PDA’s, etc.),
- Security: prevent run-time failures caused by memory limitations in critical fragments of code.
- Programming: design of new programming languages inspired from semantics,
- Fancy computer science: handle data that cannot be duplicated for physical reasons like q-bits in quantum programming.

Research Team

- Coordinators:
  - Henk Barendregt (PI)
    - ICIS, Radboud University
  - Mai Gehrke
    - IMAPP, Radboud University
  - Herman Geuvers
    - ICIS, Radboud University
  - Researchers
    - Giulio Manzonetto
      - ICIS, Radboud University
    - Jan Rutten
      - CWI & Radboud University

http://www.cs.ru.nl/calmoc