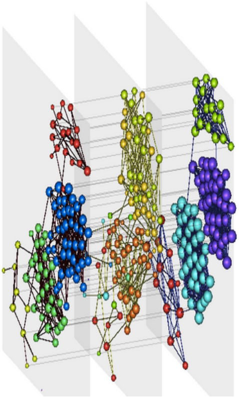
Multiplex networks analysis



source: muxviz

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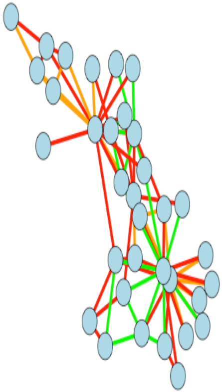
ALTERNATIVE NETWORK MODELS

Network science is mature enough to a move towards more complex, expressive models



- **Multi-relationnal networks**

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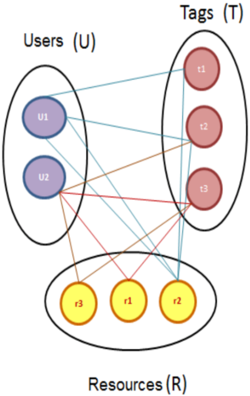
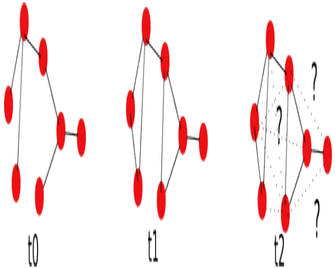


- Multi-relationnal networks
- **K-partite networks**

Alternative network models

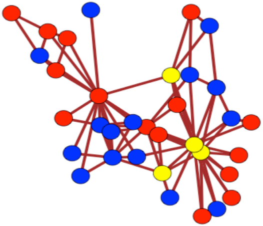
Network science is mature enough to a move towards more complex, expressive models



- Multi-relationnal networks
- K-partite networks
- **Dynamic networks**

ALTERNATIVE NETWORK MODELS

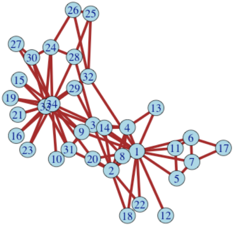
Network science is mature enough to a move towards more complex, expressive models



- Multi-relational networks
- K-partite networks
- Dynamic networks
- **Heterogeneous networks**

Alternative network models

Network science is mature enough to a move towards more complex, expressive models



- Multi-relational networks
- K-partite networks
- Dynamic networks
- Heterogeneous networks
- **Attributed networks**

ALTERNATIVE NETWORK MODELS

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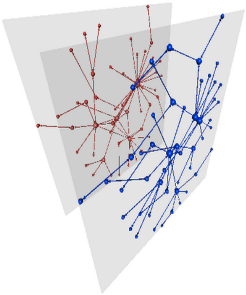
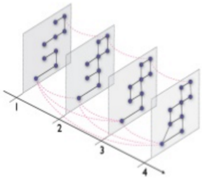


- Multi-relational networks
- K-partite networks
- Dynamic networks
- Heterogeneous networks
- Attributed networks

A powerful model : **Multiplex Network**

Multiplex Network: Definition

$G =< V, E, C >$



from [Mucha et. al., 2010]

- V set of nodes
- $E = \{E_1, \ldots, E_\alpha\} : \forall k \in [1, \alpha] E_k \subseteq V \times V$
- C Layer **Coupling** links

Coupling

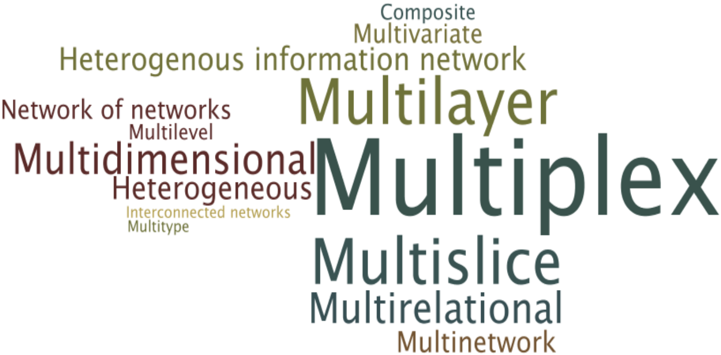
- **Ordinal Coupling** : Diagonal inter-layer links among consecutive layers.
- **Categorical Coupling** : Diagonal inter-layer links between all pairs of layers.
- Generalized coupling ? Ex. Decay functions

Notations

Notations

- $A^{[k]}$ Adjacency Matrix of slice k : $a_{ij}^{[k]} \neq 0$ si les nœuds $(v_i, v_j) \in E_k$, 0 otherwise.
- $m^{[k]} = |E_k|$. We have often $m \sim n$
- Neighbor's of v in slice k: $\Gamma(v)^{[k]} = \{x \in V : (x, v) \in E_k\}$.
- All neighbors of v : $\Gamma(v)^{tot} = \cup_{s \in \{1,\ldots,\alpha\}} \Gamma(v)^{[s]}$
- Node degree in slice k: $d_v^k = \| \Gamma(v)^{[k]} \|$
- Total degree of node v: $d_v^{tot} = ||\Gamma^{tot}(v)||$

Multiplex networks: Related terms

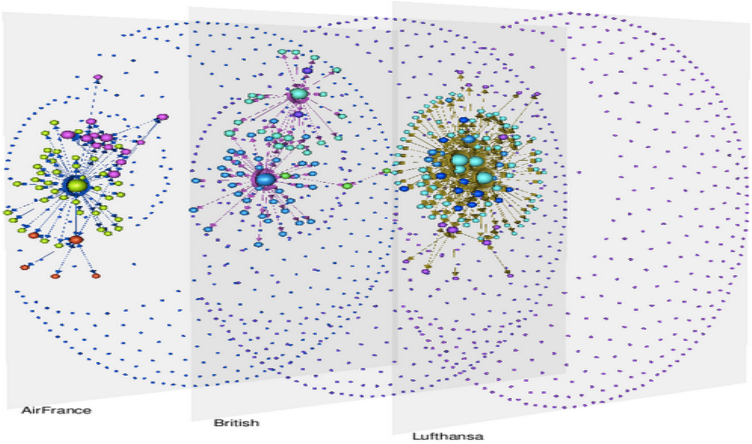


Recommended readings

- **S. Mikko Kivelä et. al.**. *Multilayer Networks*. arXiv:1309.7233, March 2014

POWER OF MULTIPLEX MODEL

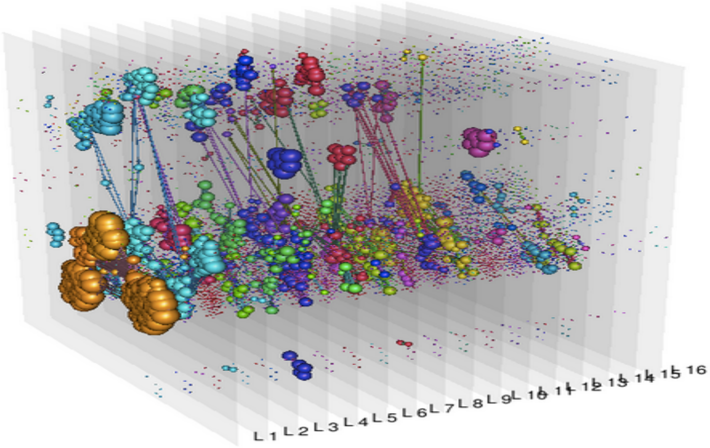
Multi-relational networks



European airports network

POWER OF MULTIPLEX MODEL

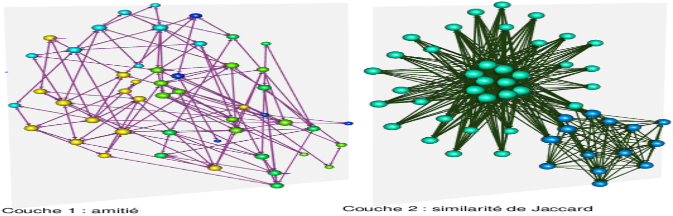
Dynamic networks



Academic collaborations per year

POWER OF MULTIPLEX MODEL

Attributed networks

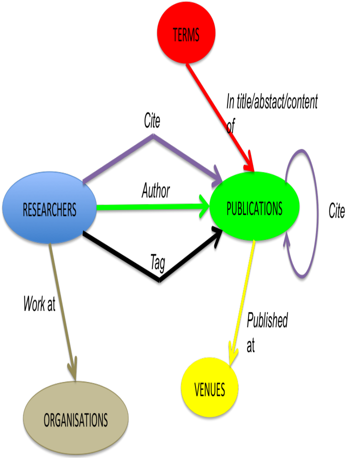


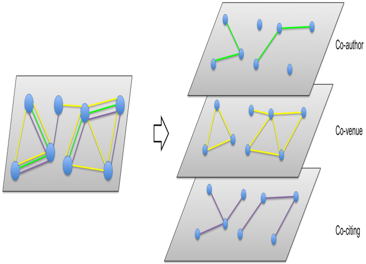
Teenage friendship network- Behavioral attributs : Sport practice level, Alcohol, Tobacco & Cannabis consumption

Proximity graphs can be defined overs nodes using attribute-similarity masures

POWER OF MULTIPLEX MODEL

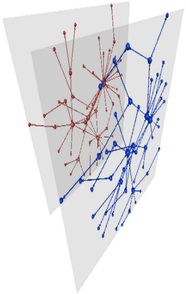
Heterogeneous networks





DBLP author-centred multiplex network

Multiplex networks : Measures

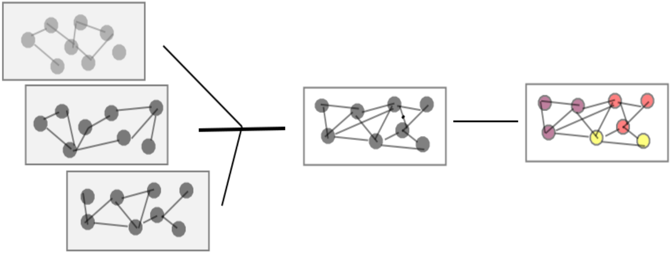


- Need of generalization of *usual measures* :
  - Degree
  - Neighbourhood
  - Centralities
  - Paths and distances
  - Clustering coefficient
  - ...
- New layer-oriented questions to answer :
  - Which layers determine the centrality of a user
  - Which layers are relevant to measure the similarity of two nodes
  - How one layer influence the evolution of another
  - ...

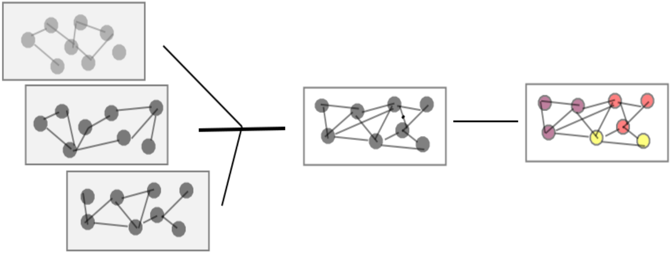
APPROACHES

1. **Transformation into a monoplex centred problem**
   - Layer aggregation approaches.
   - Hypergraph transformation based approaches
   - *Ensemble approaches*
2. **Generalization of monoplex oriented algorithms to multiplex networks**.

Layer aggregation



LAYER AGGREGATION



Aggregation functions

$$A_{ij} = \begin{cases} 1 & \exists 1 \leq l \leq \alpha : A_{ij}^{[l]} \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$A_{ij} = \frac{1}{\alpha} \sum_{k=1}^{\alpha} w_k A_{ij}^{[k]}$$

$$A_{ij} = \| \{d : A_{ij}^{[d]} \neq 0\} \|$$

$$A_{ij} = sim(v_i, v_j)$$

K-UNIFORM HYPERGRAPH TRANSFORMATION

Principle

- A k-uniform hypergraph is a hypergraph in which the cardinality of each hyperedge is exactly k
- Mapping a multiplex to a **3-uniform hypergraph** $\mathcal{H} = (\mathcal{V}, \mathcal{E})$ such that :

  $\mathcal{V} = V \cup \{1, \ldots, \alpha\}$

  $(u, v, i) \in \mathcal{E}$ if $\exists l : A_{uv}^{[l]} \neq 0, u, v \in V, i \in \{1, \ldots, \alpha\}$
- Apply hypergraphs analysis approaches (Ex. tensor-based approaches)

Multiplex: Node neighborhood

Some options

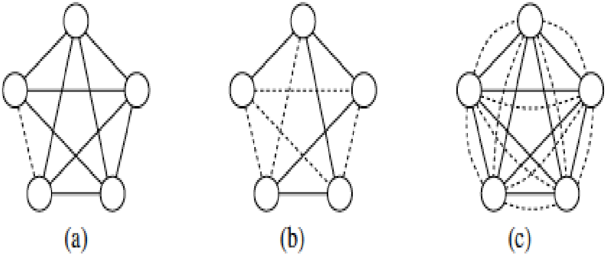
- $\Gamma^{mux}(v) = \cup_{k=1}^{\alpha} \Gamma^{k}(v)$
- $\Gamma^{mux}(v) = \cap_{k=1}^{\alpha} \Gamma^{k}(v)$
- $\Gamma^{mux}(v) = \{x \in \Gamma(v)^{tot} : sim(x, v) \geq \delta\} \; \delta \in [0, 1]$
- $\Gamma^{mux}(v) = \{x \in \Gamma(v)^{tot} : \frac{\Gamma(v)^{tot} \cap \Gamma(x)^{tot}}{\Gamma(v)^{tot} \cup \Gamma(x)^{tot}} \geq \delta\}$
- ...

PATHS, SHORTEST DISTANCE

Some options

- Path in an aggregated network
- $d_{average} = \frac{\sum_{\alpha=1}^{m} d(u,v)^{[\alpha]}}{m}$ ∀u,v $\in V$ and $(u, v) \notin E_i$.
- $path - length(u, v) =< r_1, r_2, \ldots, r_\alpha >$ where r_i number of links in layer i
- $path_x(u, v)$ dominates $path_y(u, v) \exists j : r_j^x < r_j^y, \forall k \neq j \; r_j^x \leq r_j^y$

COMMUNITY ?



What is a dense subgraph in a multiplex network ?

BerlingerioCG11

Community detection in multiplex networks

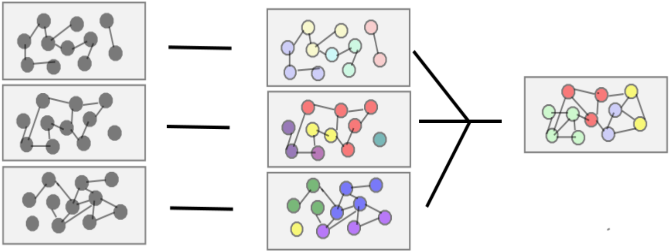
Approaches

1. **Transformation into a monoplex community detection problem**
   - Layer aggregation approaches.
   - Multi-objective optimization approach.
   - **Ensemble clustering approaches**
2. **Generalization of monoplex oriented algorithms to multiplex networks**.
   - Generalized-modularity optimization
   - Generalized info-map
   - Generalized walktrap
   - **Seed-centric approaches**

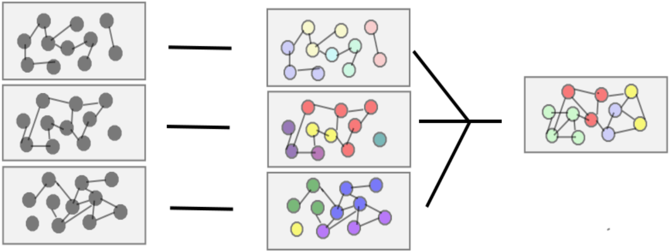
Multi-objective optimization approach [AP14]

1. Rank the set of α layers according to some *importance criteria*
2. $C_1 \leftarrow community(G^{[1]})$
3. for $i \in [2, \alpha]$ do:
   $C_i \leftarrow optimize(community(G^{[i]}), similarity(C_{i-1}))$
4. return C_α

Ensemble clustering approaches



Ensemble clustering approaches



Ensemble Clustering Strehl2003

- CSPA: Cluster-based Similarity Partitioning Algorithm
- HGPA: HyperGraph-Partitioning Algorithm
- MCLA: Meta-Clustering Algorithm
- ...

ENSEMBLE CLUSTERING: APPROACHES

CSPA: Cluster-based Similarity Partitioning Algorithm

- Let K be the number of basic models, $C_i(x)$ be the cluster in model i to which x belongs.
- Define a similarity graph on objects : $sim(v,u) = \frac{\sum_{i=1}^{K} \delta(C_i(v), C_i(u))}{K}$
- Cluster the obtained graph :
  - Isolate connected components after prunning edges
  - Apply community detection approach
- Complexity : $\mathcal{O}(n^2kr)$: n # objects, k # of clusters, r# of clustering solutions

Multiplex Modularity

Generalized modularity mucha2010community

- $$Q_{multiplex}(P) = \frac{1}{2\mu} \sum_{c \in P} \sum_{\substack{i,j \in c \\ k,l:1 \to \alpha}} \left(\left(A_{ij}^{[k]} - \lambda_k \frac{d_i^{[k]} d_j^{[k]}}{2m^{[k]}} \right) \delta_{kl} + \delta_{ij} C_{ij}^{kl} \right)$$
- $\mu = \sum_{\substack{j \in V \\ k,l:1 \to \alpha}} m^{[k]} + C_{jk}^{l}$
- C_{ij}^{kl} Inter slice coupling = 0 $\forall i \neq j$

Seed-centric algorithms [Kan14]

Algorithm 3 General seed-centric community detection algorithm

Require: $G =< V, E >$ a connected graph,
1: $\mathcal{C} \leftarrow \emptyset$
2: $S \leftarrow$ **compute_seeds(G)**
3: **for** $s \in S$ **do**
4: $\quad C_s \leftarrow$ **compute_local_com(s,G)**
5: $\quad \mathcal{C} \leftarrow \mathcal{C} + C_s$
6: **end for**
7: **return compute_community($\mathcal{C}$)**

THE LICOD ALGORITHM [YK14]

1. Compute a set of seeds that are likely to be leaders in their communities

   Heuristic : nodes having higher **degree centralities** *than their neighbors*

2. Each node in the graph ranks seeds in function of its own preference

   In function of increasing **Shortest path**

3. Iterate till convergence: Each node modifies its preference vector in function of **neighbor's** preferences

   Applying **rank aggregation** *methods.*

MuxLicod

Multiplex degree centrality [BNL13]

$$d_i^{multiplex} = -\sum_{k=1}^{\alpha} \frac{d_i^{[k]}}{d_i^{[tot]}} log\left(\frac{d_i^{[k]}}{d_i^{[tot]}}\right)$$

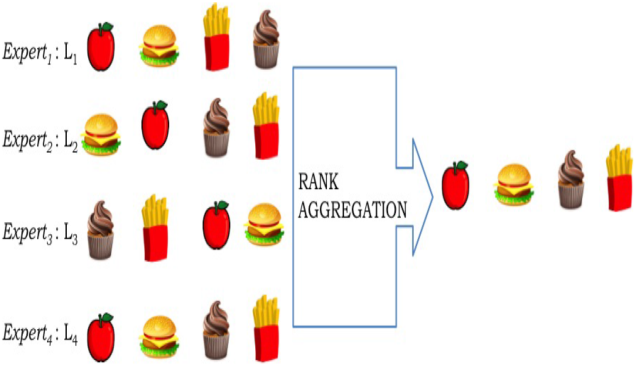
Multiplex shortest path

$$SP(u,v)^{multiplex} = \frac{\sum_{k=1}^{\alpha} SP(u,v)^{[k]}}{\alpha}$$

Multiplex neighborhood

$$\Gamma^{mux}(v) = \{x \in \Gamma(v)^{tot} : \frac{\Gamma(v)^{tot} \cap \Gamma(x)^{tot}}{\Gamma(v)^{tot} \cup \Gamma(x)^{tot}} \geq \delta\}$$

RANK AGGREGATION



[PK12, DKNS01]

Other algorithms

1. Random walk based approach (Generalization of Walktrap [KM15]
2. Generalized infomap [DLAR15]

EVALUATION CRITERIA I

1. Multiplex modularity
2. Redundancy [BCG11]

$$\rho(c) = \sum_{(u,v)\in \bar{\bar{P}}_c} \frac{\| \{k : \exists A_{uv}^{[k]} \neq 0\} \|}{\alpha \times \| P_c \|}$$

$\bar{\bar{P}}$ the set of couple (u, v) which are directly connected in at least two layers

3. Complementarity :$\gamma(c) = \mathcal{V}_c \times \varepsilon_c \times \mathcal{H}_c$

Evaluation criteria II

- Variety $\mathcal{V}_c$: the proportion of occurrence of the community c across layers of the multiplex.

$$\mathcal{V}_c = \sum_{s=1}^{\alpha} \frac{\|\exists (i,j) \in c / A_{ij}^{[s]} \neq 0\|}{\alpha - 1} \tag{2}$$

- Exclusivity ε_c : number of pairs of nodes, in community c, that are connected exclusively in one layer.

$$\varepsilon_c = \sum_{s=1}^{\alpha} \frac{\|\overline{P_{c,s}}\|}{\|P_c\|} \tag{3}$$

Evaluation criteria III

- Homogeneity $\mathcal{H}_c$: How uniform is the distribution of the number of edges, in the community c, per layer.

$$\mathcal{H}_c = \begin{cases} 1 & if \quad \sigma_c = 0 \\ 1 - \frac{\sigma_c}{\sigma_c^{max}} & otherwise \end{cases} \tag{4}$$

with

$$avg_c = \sum_{s=1}^{\alpha} \frac{\|P_{c,s}\|}{\alpha}$$

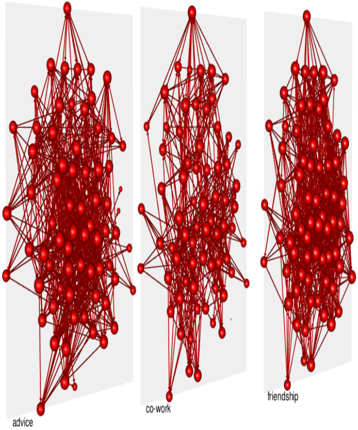
$$\sigma_c = \sqrt{\sum_{s=1}^{\alpha} \frac{(\|P_{c,s}\| - avg_c)^2}{\alpha}}$$

$$\sigma_c^{max} = \sqrt{\frac{(max(\| P_{c,d} \|) - min(\| P_{c,d} \|))^2}{2}}$$

DATASETS

Benchmark networks

- Lazzega Lawyer network
  - #nodes 71
  - #layer 3



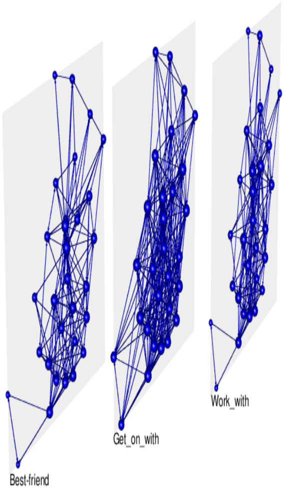
DATASETS

Dataset

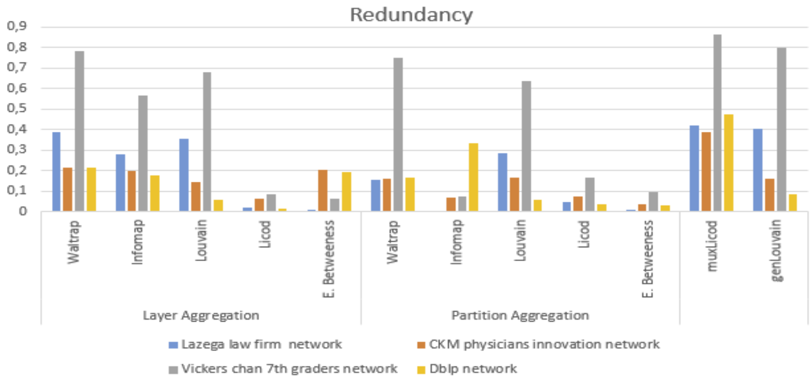
Physicians collaboration network

#nodes 246

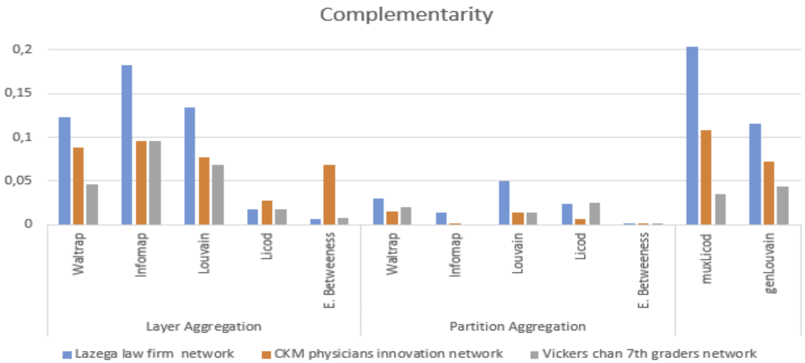
#layers 3



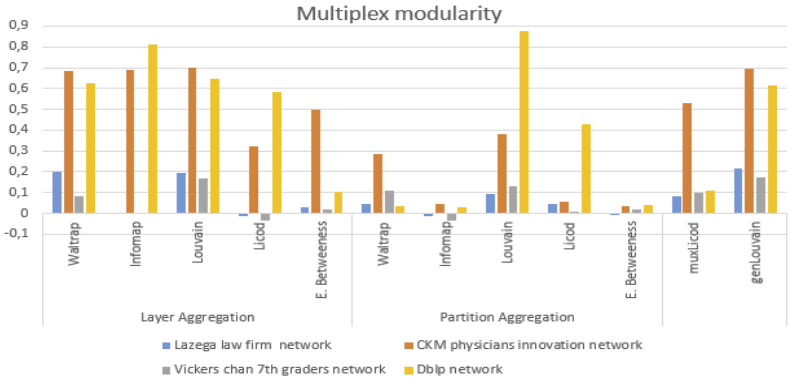
Results: Redundancy



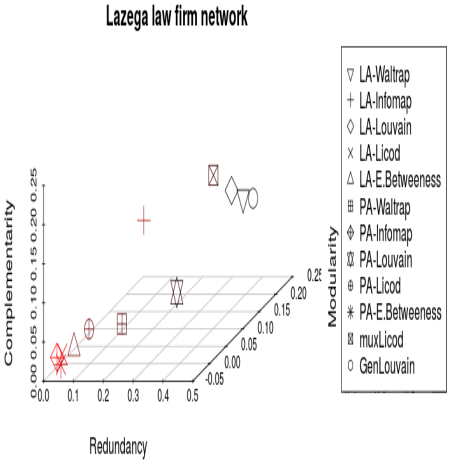
RESULTS: COMPLEMENTARITY

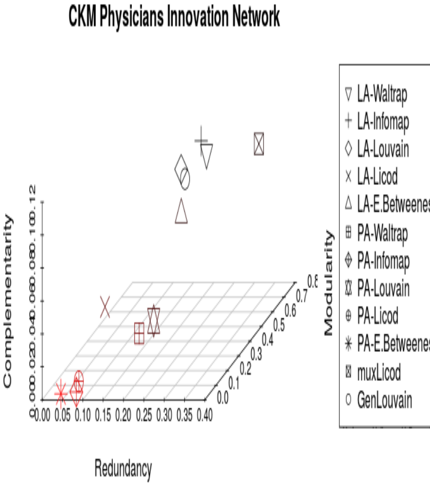


Results: multiplex modularity



PARETO FRONT





LAZEGA DATASET: COMPARATIVE STUDY

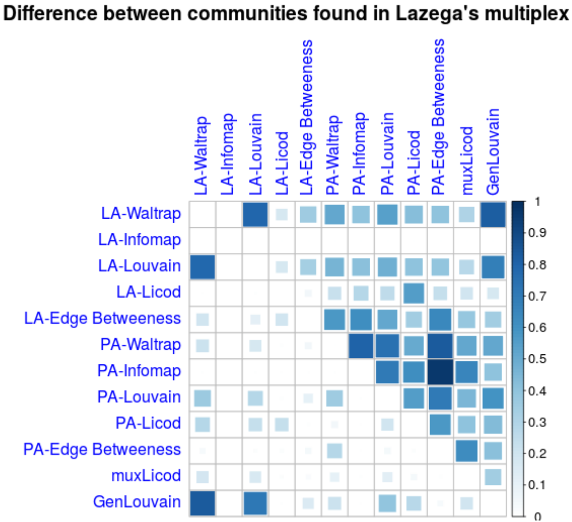


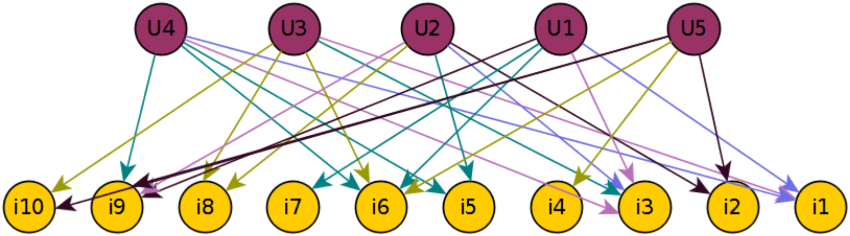
Figure: NMI (lower triangular part) , adjusted Rand (upper triangular part).

APPLICATIONS

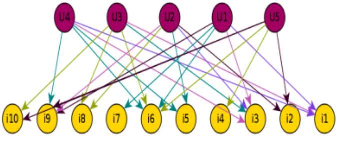
1. Film recommandation
2. Tag recommendation
3. Collaboration recommendation
4. Ensemble clustering selection

FILM RECOMMENDATION

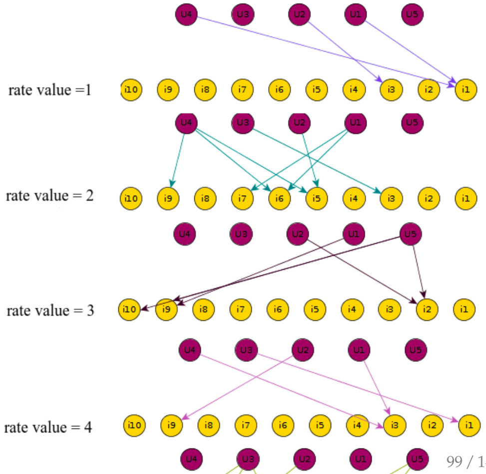
Film rating matrix = bipartite graph



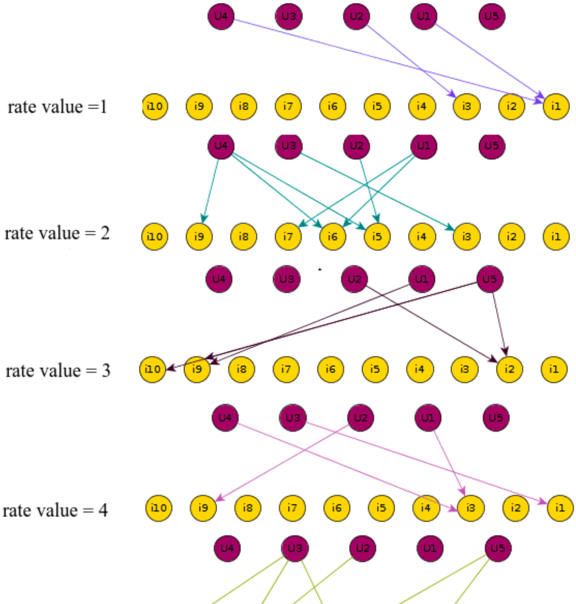
FILM RECOMMENDATION

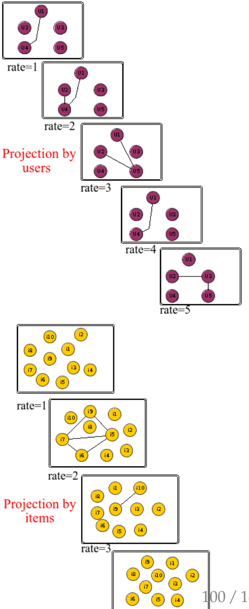


Step 1 : Dissociate each rate value
from the bipartite graph



Film recommendation





FILM RECOMMENDATION : MULTIPLEX NETWORK

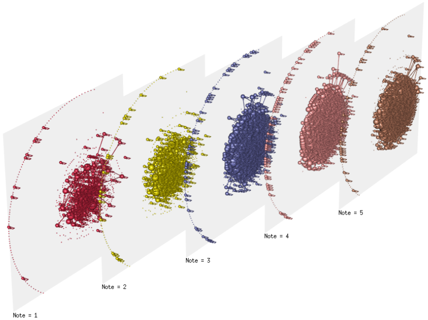


Figure: Movieslens 100k multiplex (Projection by users)

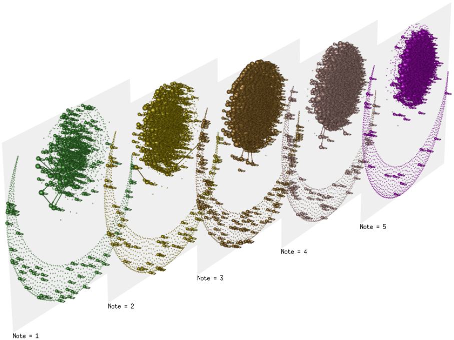


Figure: Movieslens 100k multiplex (Projection by movies)

Film recommendation : results

Simple approach

Recommend the statistical mode value of links linking clusters of target user to the cluster of target films

	MAE	RMSE	Precision	Recall	F1-measure
GTM	0.9441	1.2549	0.2185	0.2207	0.2195
T. co-clustering	0.9293	1.2562	0.25587	0.2094	0.2303
muxlicod	0.9635	1.2773	0.2274	0.2134	0.2202
LA louvain	0.8352	1.1509	**0.3113**	**0.2521**	**0.2779**
LA walktrap	**0.8216**	**1.1155**	0.2642	0.2233	0.2420
PA louvain	0.8713	1.1917	0.2532	0.2032	0.2245
PA walktrap	0.8801	1.2023	0.2705	0.2011	0.2283

Table: Result of the proposed recommendation system with each algorithm in MovieLens 100k dataset (PA : Partition Aggregation, LA : Layer Aggregation

APPLICATION II: TAG RECOMMENDER (TLTR)

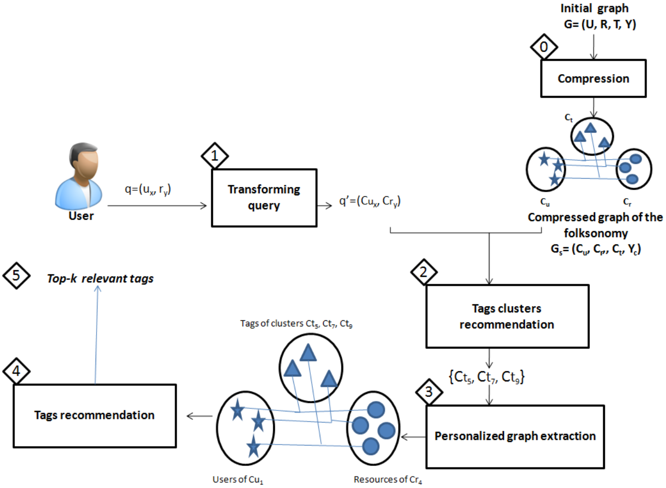


Figure: TLTR model

Folksonomy graphs

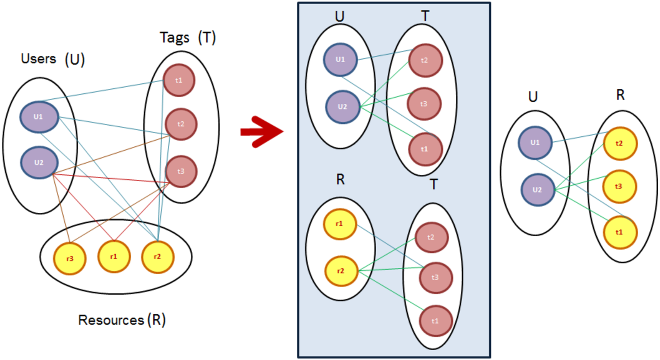


Figure: Tripartite graph projection into three bipartite graphs

Multiplex network construction

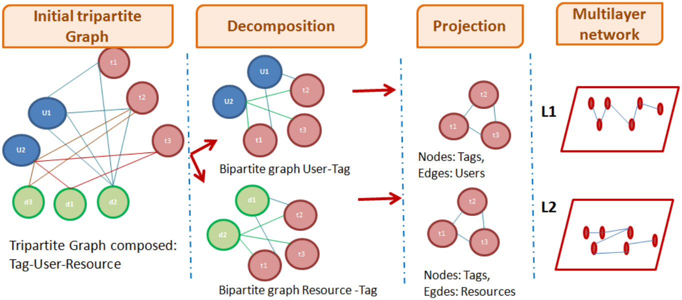


Figure: Tag multiplex network: Steps of transformation

Experiments: Bibsomnomy dataset

# Users	# Tags	# Resources	# Edges
116	412	361	24297

Table: Bibsonomy dataset

Networks	slices	Nodes	Edges	Density
User	*User-Resource*	116	901	0,135
	User-Tag	116	985	0,147
Tag	*Tag-Resource*	412	2496	0,0294
	Tag-User	412	1956	0,0231
Resource	*Resource-Tag*	361	2814	0,0433
	Resource-User	361	1685	0,0259

Table: Multiplex networks of Bibsonomy

TAG RECOMMANDATION: RESULTS

Graphs	#Nodes	#Edges	#Users	#Tags	#Resources
G	889	24297	116	412	361
G_c (Mux-Licod)	434	1677	97	154	183
compression in %	$51,18$	$93,1$	$16,37$	$62,62$	$49,30$
G_c (GenLouvain)	16	79	4	6	6
compression in %	$98,2$	$99,67$	$96,55$	$98,54$	$98,33$
G_c (LA (Licod))	91	46	13	40	38
compression in %	$89,76$	$99,81$	$88,79$	$90,29$	$89,47$
G_c (LA (Louvain))	9	27	3	3	3
compression in %	$98,98$	$99,88$	$97,41$	$99,27$	$99,16$
G_c (EC (Licod))	151	993	3	89	59
compression in %	$83,08$	$95,91$	$97,41$	$78,39$	$83,65$
G_c (EC (Louvain))	25	187	8	11	6
compression in %	$97,18$	$78,96$	$93,10$	$97,33$	$98,33$

Tag recommandation: results

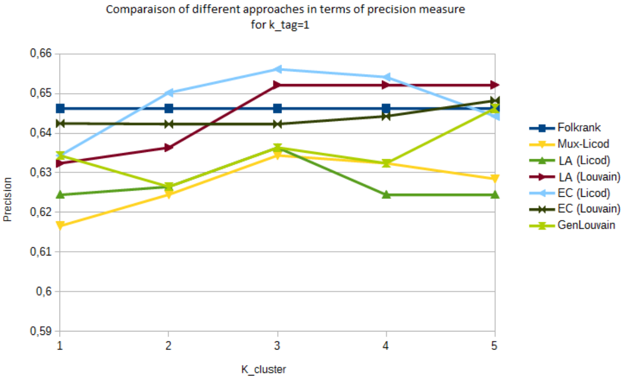


Figure: Comparative study of different tags recommendation approaches in terms of precision with $k_t = 1$

TAG RECOMMANDATION: RESULTS

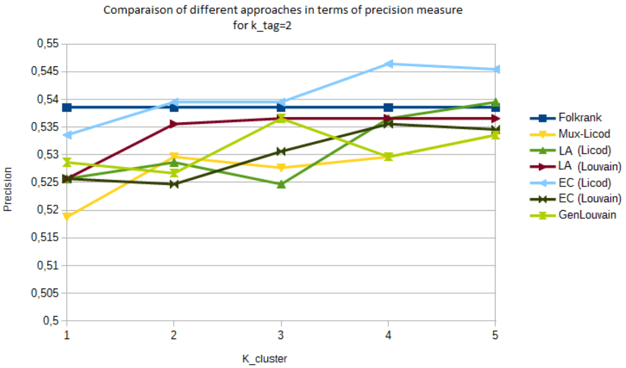


Figure: Comparative study of different tags recommendation approaches in terms of precision with $k_t = 2$

Tag recommandation: results

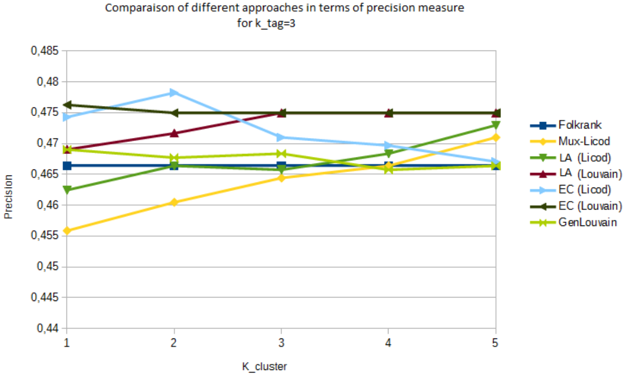


Figure: Comparative study of different tags recommendation approaches in terms of precision with $k_t = 3$

Tag recommandation: results

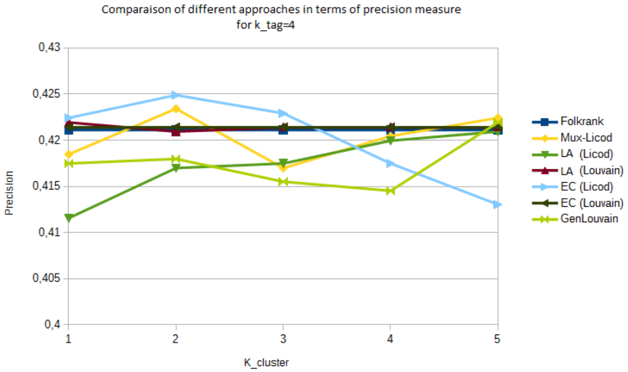
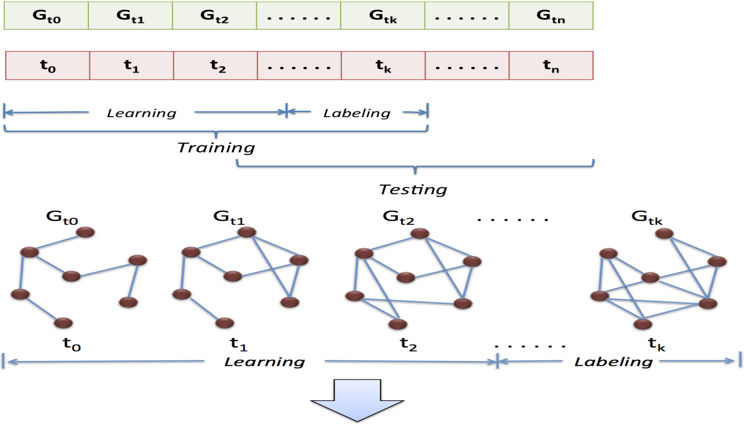


Figure: Comparative study of different tags recommendation approaches in terms of precision with $k_t = 4$

Discussion

- Multiplex approaches outperform layer aggregation and EC approaches on benchmark networks
- Layer aggregation approaches do well for film recommendation !
- EC approaches rank first for Tag recommendation !
- **Problem what is the Validity of topological community quality indexes ?**

Application III: Scientific Collaboration recommender

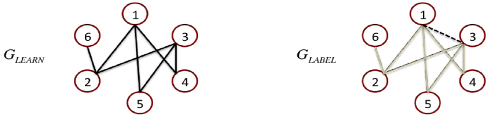


$$G_{learn} = \bigcup_{t=t_0}^{t_{k-2}} G_t \qquad G_{label} = \bigcup_{t=t_{k-1}}^{t_k} G_t$$

LINK PREDICTION: SUPERVISED APPROACH

Application of supervised machine learning algorithms

- Work of [Hasan & al., 2006]



Examples	$CN = \|\Gamma(x) \cap \Gamma(y)\|$	$JC = \frac{\|\Gamma(x) \cap \Gamma(y)\|}{\|\Gamma(x) \cup \Gamma(y)\|}$	*Class*
(1 , 3)	3	1	Positive
(1 , 6)	1	0.33	Negative
(2 , 4)	2	0.66	Negative
(2 , 5)	2	0.66	Negative
(3 , 6)	1	0.33	Negative
(4 , 5)	2	1	Negative
(4 , 6)	0	0	Negative
(5 , 6)	0	0	Negative

Experiments: DBLP

Years	Properties	Co-Author	Co-Venue	Co-Citation
1970-1973	*Nodes*	91	91	91
	Edges	116	1256	171
1972-1975	*Nodes*	221	221	221
	Edges	319	5098	706
1974-1977	*Nodes*	323	323	323
	Edges	451	9831	993

Table: Basic statistics about the 3-layer DBLP multiplex networks

Years		# Positive	# Negatives
Train/Test	Labeling		
1970-1973	1974-1975	16	1810
1972-1975	1976-1977	49	12141
1974-1977	1978-1979	93	26223

Table: # examples extracted from co-authorship layer (number of unconnected nodes in connected components)

Link prediction: Results

Attributes	Learning:1970-1973 Test:1972-1975		Learning:1972-1975 Test:1974-1977	
	F-measure	**AUC**	**F-measure**	**AUC**
Set_{direct}	0.0357	0.5263	0.0168	0.4955
$Set_{direct+indirect}$	0.0256	0.5372	0.0150	0.5132
$Set_{direct+multiplex}$	0.0592	0.5374	0.0122	0.5108
Set_{all}	0.0153	0.5361	0.0171	0.5555
$Set_{multiplex}$	0.0374	0.5181	0.0185	0.5485

Table: Comparative link prediction results applying decision tree algorithm using different types of attributs

Application IV: Ensemble Clustering Selection

Motivation

The quality of a consensus clustering depends on both the **quality** and **diversity** of input base clusterings [FL08, AF09, NCC13, ADIA15].

Problem definition

- Let $\Pi = \{\pi_1, \ldots, \pi_n\}$ be a set of base partitions
- $\mathcal{ES}(\Pi) = \Pi^* \subset \Pi : \mathcal{Q}(EC(\Pi^*)) > \mathcal{Q}(EC(\Pi))$
- $\mathcal{Q}$: Quality of the consensus clustering

Diversity

Clustering Similarity measures

- Purity
- Rand/ARI
- NMI (Normlized mutual information)
- IV (Information variation) [Mei03]
- ...

QUALITY

Cluster internal quality indexes [AR14]

- Silhouette index,
- Calinski-Harabasz index
- Davis-Bouldin index
- Dunn index
- ...

Network-oriented indexes

- Modularity
- Average conductance
- Average local Modularities : L, M, R [Kan15]
- See also [YL12]
- ...

Ensemble selection approaches : limitations

- Existing approaches are defined for attribute/value datasets with metric distances
- Use of one quality/diversity measure.
- Requires the number of clusters to select as input.
- ...

Proposed approach: contributions

- Designed for both networks and attribute/value datasets
- Use of an *ensemble* of quality/diversity measures.
- The number of selected base clustering is automatically computed.

Ensemble selection approach

The idea

- Cluster the set of base clusterings using an ensemble of similarity measures

  Apply a **multiplex community detection** *algorithm to a multiplex network whose nodes are the set of base clusterings and whose layers are defined by a set of* **proximity graphs**, *each defined according a to a given similarity measure*

- From each cluster select the node (i.e clustering) that is ranked first according to an ensemble of quality measures.

  Apply **ensemble ranking** *algorithms*

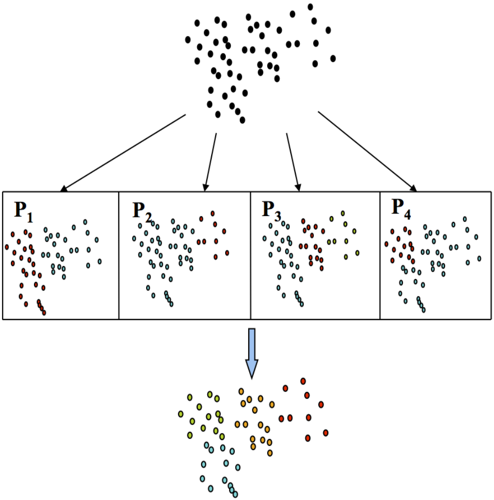
Ensemble selection approach

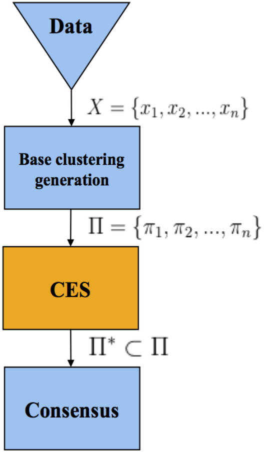
Algorithm 4 Graph-based cluster ensemble selection algorithm

Require: $\Pi = \{\pi_1, \ldots, \pi_r\}$ a set of base clusterings
Require: $\mathcal{S} = \{S_1, \ldots, S_n\}$ A set of partition similarity functions
Require: $\mathcal{Q} = \{Q_1, \ldots, Q_m\}$ A set of partition quality functions

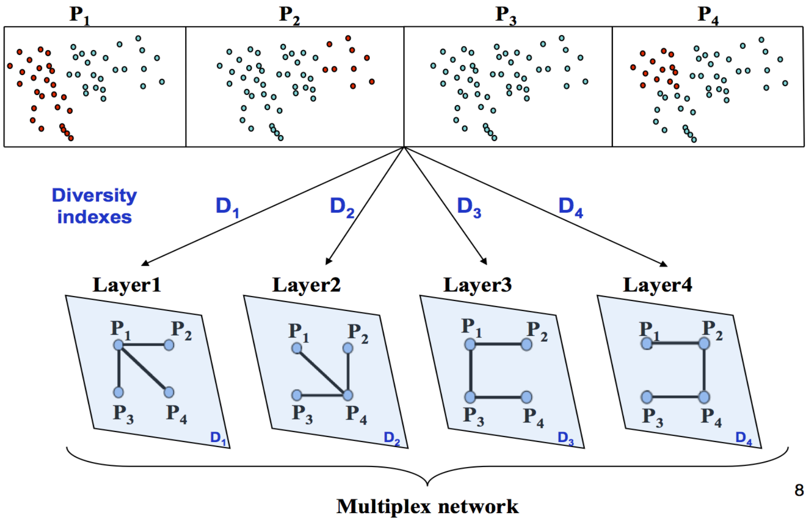
$\Pi^* \leftarrow \emptyset$
$MUX \leftarrow$ **Multiplex(**Π**)**
for all $S_i \in \mathcal{S}$ **do**
    MUX.**add_layer**(proximity_graph(Π, S_i))
end for
$\mathcal{C} = \{c_1, \ldots, c_k\} \leftarrow$ **community_detection**(MUX)
for all $c \in \mathcal{C}$ **do**
    $\hat{\pi} \leftarrow$ **ensemble_Ranking**(c, $\mathcal{Q}$)
    $\Pi^* \leftarrow \Pi^* \cup \{\hat{\pi}\}$
end for
return Π^*

THE PROPOSED APPROACH

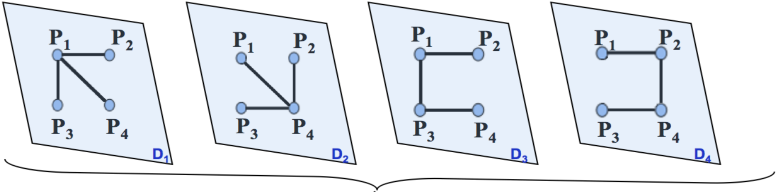




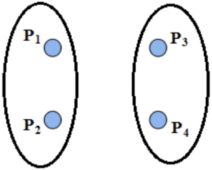
The proposed approach



THE PROPOSED APPROACH



Community detection in multiplex network



Communities

Qualité (Rang)

Q	I_1	I_2	I_3	TOT
P_1	1	2	1	<u>4</u>
P_2	2	1	2	5

Q	I_1	I_2	I_3	TOT
P_3	1	2	1	<u>4</u>
P_4	2	1	2	5

P_1 P_3

Selection

9

Ensemble ranking

Problem

- Let L be a set of elements to rank by n rankers
- Let σ_i be the rank provided by ranker i
- **Goal: Compute a consensus rank of** L.

Déjà Vu: Social choice algorithms, but ...

- Small number of voters and big number of candidates
- Algorithmic efficiency is required

Algorithms

- Borda
- Kemeny approaches (commuting Condorcet winner if it exists)

Experiment on small networks with known ground truth partitions

- Generation of 20 base clusterings applying a standard Label propagation algorithm
- Proximity graphs : RNG
- $\mathcal{S}$ = { NMI, ARI, VI } $\mathcal{Q}$ = { modularity, Local modularities L, M, R }

Table: Evaluation of the proposed graph-based ensemble selection

Dataset	Approach	NMI	ARI
Zachary	Ensemble clustering without selection	0.57	0.46
	Ensemble clustering with selection	**0.77**	0.69
US Politics	Ensemble clustering without selection	0.55	0.68
	Ensemble clustering with selection	**0.68**	0.67
Dolphins	Ensemble clustering without selection	0.55	0.39
	Ensemble clustering with selection	**0.58**	0.59

Experiment II : DBLP co-authorship network

- Co-authorship network 1970-1977 (GCC) : $|V| = 643$, $|m| = 886$
- Generation of 10, 100 base clusterings
- Proximity graphs : RNG
- $\mathcal{S} = \{$ NMI, ARI, VI $\}$ $\mathcal{Q} = \{$ modularity, Local modularities L, M, R $\}$

Table: Evaluation of the proposed graph-based ensemble selection

# base clusterings	10
Nodes Compression without selection	18,3%
Nodes Compression with selection	20,9%
Edge compression without selection	17,2%
Edge compression with selection	17,6%
Modularity without selection	0.3734
Modularity with selection	**0.43756**

Experiment II : DBLP co-authorship network

Table: Evaluation of the proposed graph-based ensemble selection

# base clusterings	100
Nodes Compression without selection	35,1%
Nodes Compression with selection	40,3%
Edge compression without selection	36,2%
Edge compression with selection	38,3%
Modularity without selection	0.4031
Modularity with selection	**0.4665**

MUXVIZ

- *R* package
- Main features :
  - Visualization
  - Layer compression methods
  - Basic metrics
  - Community detection : Modularity-based, infomap
- Input : text file per layer + one file for the general structure.

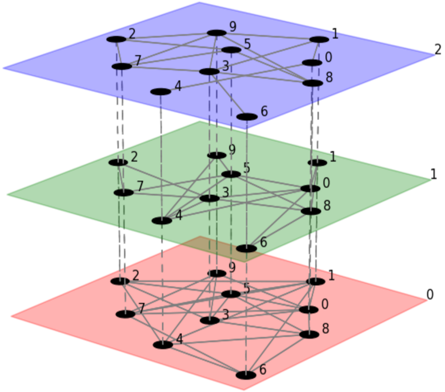
MUNA

- Available for *R* and *Python*
- Built on top of *igraph*
- Extended set of multiplex network edition functions (similar to igraph)
- Basic metrics : degree, neighborhood
- Extended set of community detection approaches
- Topological community evaluation indexes.
- Limitations :
  - *No visualisation support*
  - Simple categorical coupling only.

PYMNET

- Pure Python + integration with networkX package.
- Can handle general multilayer networks
- Rule based generation and lazy-evaluation of coupling edges
- Various network analysis methods, transformations, reading and writing networks, network models etc.
- Visualization support

Pymnet: Visualisation exemple



CONCLUSIONS

- **Multiplex networks** provide a rich representation of real-world interaction systems
- A lot of work to reformulate basic network concepts for multiplex settings
  ex. Roles, RandomWalk, PageRank, etc.
- New tools for multiplex mining : Muna [FK15], muxviz[**?**], *Pymnet*
- Community evaluation: still an open problem
- Uncovered topics : Layer selection and compression, Co-evolution models, Dynamics on multiplex networks
- Ideas under exploration:

  Multiplex approach for attributed networks mining
  Multiplex of multiplexes
  Interactive Multiplex network visualisation.
  Benchmarking available tools