

Internship proposal: Randomised strategies and 0-1 law

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Keywords. combinatorics, tree generation, rewriting, probabilities, random walks, λ -calculus

Prerequisite. Bases of probability, combinatorics and rewriting. Expertise is at least one of those more advanced topics: random walks, tree generation, or λ -calculus.

Principle

Given a binary tree, we define a reduction step as choosing a node $n(l, r)$ randomly and uniformly, then duplicating this node into $n(n(l, r), n(l, r))$. By reproducing this step, we define an algorithm that generates randomly a binary tree. We would like to understand the mean behaviour of this tree. In particular, we conjecture that this tree is growing on average in $n \cdot \log(n)$.

This result will lead to a fun application in rewriting theory: orthogonal TRSs, when applied a random strategy (redex chosen uniformly at each step), would then respect a 0-1 law, in the sense that any term would have a probability of convergence either 0 or 1.

Going further, we can look at the, more complex, situation of the λ -calculus that is conjectured to verify this 0 – 1 law. This is a conjecture from Panengaden running for 5-10 years and considered difficult.

References

- A first note on the general cases.
- A second note on combinatorial details.